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The Application of Robust Regression to a Production Function Comparison

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Abstract: The adequate representation of crop response functions is crucial for agronomic as well as agricultural economic modeling and analysis. So far, the evaluation of such functions focused on the comparison of different functional forms. In this article, the perspective is expanded also by considering different regression methods. This is motivated by the fact that exceptional crop yield observations (outliers) can cause misleading results if least squares regression is applied. In order to address this problem we also apply robust regression techniques that are not affected by such outliers. We evaluate the quadratic, the square root and the Mitscherlich-Baule function using the example of Swiss corn (*Zea mays* L.) yields. It shows that the use of robust regression narrows the range of optimal input levels across different functional forms and reduces potential costs of misspecification compared to least squares estimation. Thus, differences between functional forms are reduced by applying robust regression.

Keywords: Production function estimation, production function comparison, robust regression, crop response.

INTRODUCTION

The adequate representation of production or crop yield functions is crucial for modeling purposes in agronomic, agricultural and environmental economic analyses. The discussion and estimation of different functional forms has therefore gained much attention in agronomic and agricultural economics literature. Various functional forms have been considered so far, but less attention has been given to the estimation techniques in general and the impact of exceptional crop yield observations (outliers). The latter is important since the Least Squares (LS) fitting criterion can produce misleading results if data sets contain outliers, such as exceptional yields caused by extreme weather events or climate situations. In order to address this problem we apply robust regression techniques that are not affected by such outliers. The aim of this article is to analyze the influence of estimation techniques on the evaluation of different functional forms that describe crop responses. This extends the literature on the comparison of different functional forms [e.g. 1-5] by taking the effect of outliers for the estimation and evaluation of crop production functions into account.

So far, comparison of functional forms has been based on the coefficient of determination [2], residual distribution [3], non-nested hypothesis testing [4] and potential misspecification costs [5], respectively. Using LS and robust regression, we devote special attention to the cost of misspecification which constitutes an economic approach to the comparison of production functions. This allows us to assess the potential underestimation of net revenues that would arise from using calculations based on LS instead of robust regression methods or from an improper specification of the functional form.

We apply a meta-modeling approach that makes use of crop yield data generated with a biophysical simulation model to estimate and compare crop production functions. Biophysical simulation allows us to generate an enlarged data base compared with field observations. It particularly enables the creation of comprehensive data sets of crop yields with respect to the variation of analyzed factors such as agricultural inputs, while keeping other factors constant. The resulting data set is used to estimate different types of crop production functions. Those are subsequently integrated in a non-linear economic optimization model to assess optimal factor inputs, such as nitrogen fertilizer and irrigation water. Numerical examples are given for Swiss corn (*Zea mays L.*) yields.

MATERIAL AND METHODOLOGY

Production Functions

Three types of crop production functions are analyzed in this study: two polynomial specifications (the quadratic and the square root function) and the Mitscherlich-Baule function. These functional forms are frequently used in the literature and proved to accurately capture the underlying relationships [1, 4-12].

Being aware that corn yields are driven by numerous factors, we focus our analysis on two crucial production factors: nitrogen fertilizer and irrigation water. Following Llewelyn and Featherstone [5], production functions are used to describe corn yield responses to nitrogen and irrigation water in a simple analytical description, which is necessary to represent yield response processes in agricultural and environmental economic allocation models. These functions implicitly consider other production factors such as soil and climate [13]. In contrast, complex production functions — e.g. including sets of climate variables and their interactions with management variables — can complicate or preclude straightforward application in further economic analysis. Therefore, we focus in this study on simple analytical forms

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of production functions that are widely used in practice [e.g. 8, 11-17].

The *quadratic form*, shown in equation (1), consists of an additive composition of the input factors, their squared values, and an additional interaction term. The latter elucidates whether the input factors are independent of each other or not. The quadratic function is formally defined as follows:

$$Y = \alpha_0 + \alpha_1 \cdot N + \alpha_2 \cdot W + \alpha_3 \cdot N^2 + \alpha_4 \cdot W^2 + \alpha_5 \cdot N \cdot W \quad (1)$$

Y denotes corn yield per area, *N* the amount of inorganic nitrogen applied, and *W* irrigation water applied. The α_i's are parameters that must satisfy the subsequent conditions in order to ensure decreasing marginal productivity of each input factor: $\alpha_1, \alpha_2 > 0$ and $\alpha_3, \alpha_4 < 0$. Furthermore, if $\alpha_5 > 0$ the two input factors are complementary. They are competitive if $\alpha_5 < 0$, while $\alpha_5 = 0$ indicates independence of the two input factors.

The *square root function* (equation 2) is very similar to the quadratic form but produces different shapes of the curves. The square root form is defined as follows:

$$Y = \alpha_0 + \alpha_1 \cdot N^{1/2} + \alpha_2 \cdot W^{1/2} + \alpha_3 \cdot N + \alpha_4 \cdot W + \alpha_5 \cdot (N \cdot W)^{1/2} \quad (2)$$

To ensure decreasing marginal productivity of each input factor, the parameters must satisfy the same conditions as for the quadratic form, and their interpretation is identical.

The *Mitscherlich-Baule function* (equation 3) allows for a growth plateau, which follows from the von Liebig approach to production functions. Moreover, this functional form is characterized by continuously positive marginal productivities of the input factors. It does not exhibit negative marginal productivities, as the above polynomial forms. Formally, the Mitscherlich-Baule function is given by

$$Y = \alpha_1 \cdot (1 - \exp(-\alpha_2 \cdot (\alpha_3 + N))) \cdot (1 - \exp(-\alpha_4 \cdot (\alpha_5 + W))) \quad (3)$$

with α_1 representing the growth plateau, and α_3 and α_5 the natural input endowments, that include nitrogen in the soil (α_3) and water endowments (α_5) such as soil moisture. The coefficients α_2 and α_4 describe the influence of the corresponding input factors on the yield. Unlike the classical von Liebig production function, the Mitscherlich-Baule function allows for factor substitution. It is not linear limitational in the input factors as the von Liebig function, i.e. the isoquants are not right-angled.

Data

Our analysis and estimation of production functions is based on simulated corn yield data that is generated with the CropSyst model. This is a deterministic crop yield simulation model that has been widely used and validated (see Stöckle *et al.* [18], for a review of studies using CropSyst). It involves various above and below ground processes, such as soil water budget, soil-plant nitrogen budget, crop phenology, canopy and root growth, biomass production, crop yield, residue production and decomposition, and soil erosion by water. These processes are simulated with daily time step. The model is calibrated to field trials and sample data. Model settings and calibration for the Swiss Plateau region are presented in Torriani *et al.* [19].

In our analysis, CropSyst is driven by daily weather data from six different locations on the Swiss Plateau for the years 1981 - 2003, as provided by the Swiss Federal Office of Meteorology and Climatology (MeteoSwiss). These locations are distributed over the eastern Swiss Plateau ranging from 06°57' to 08°54' longitude and are located at elevation levels between 422 and 565 meter above sea level [20]. Compared to an approach with one single location, the use of observations from six different weather stations broadens the database and allows us to represent production functions for a large proportion of the entire Swiss corn producing acreage. Growing season average temperatures and precipitation sums (average over the six locations) for the period 1981-2003 are shown in Fig. (**1**).

To enable meta-modeling analysis and avoid distortions due to dynamic effects, all simulations are conducted using identical starting conditions. Accordingly, the simulation and subsequent data analysis are restricted to one uniform type of soil for all locations, characterized by texture with 38% clay, 36% silt, 26% sand and soil organic matter content at 2.6% weight in the top soil layer (5 cm) and 2.0% in lower soil layers [19]. Moreover, the type of management is uniform for all simulations. Identical seeding dates, irrigation settings (possible from day one after sowing to harvesting, never exceeding field capacity), fertilizer type (inorganic nitrogen fertilizer) and fertilizer application dates are used in CropSyst [20]. This approach avoids distortions due to non-uniform soil and management properties.

To have a comprehensive data set, one simulation is conducted without application of fertilizer and irrigation for each location and each year. Additional combinations of irrigation and fertilizer are generated randomly. Taking nitrogen fertilizer application rates from 0 to 320 kg/ha and irrigation water from 0 to 340 mm, this results in 212 different levels of nitrogen application to the plants and 60 different levels of irrigation.

The resulting dataset consists of 527 observations. Assuming a dry matter content of 85%, average yields for three different ranges of irrigation *W* and fertilizer *N* application, respectively, are shown in Table **1**. This rough approximation of the average corn yields reveals a global yield maximum for $71 \le W \le 140$ and $76 \le N \le 150$. Simulated corn yields decrease if the amounts of irrigated water or applied fertilizer deviate from those input ranges.

In our meta-modeling approach, output of the biophysical model is restructured into crop production functions. As a consequence, key relationships among the factors studied that are relevant for aggregate economic analysis can be isolated on a yearly basis [10]. In contrast, processes in the biophysical model are conducted on a daily time step. Thus, the relationships estimated in the crop production functions do not replicate settings in the biophysical model, i.e. in the data generating process. Similar meta-modeling approaches that combine biophysical simulations and economic modeling by using production functions are used, for instance, by Jalota *et al.* [10], and Llewelyn and Featherstone [5].

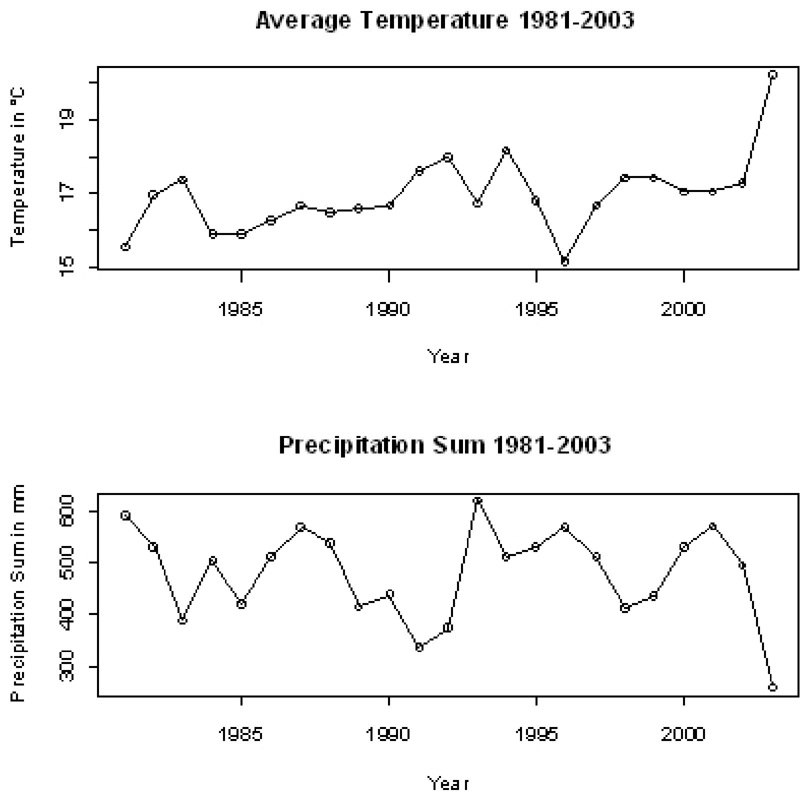

Fig. (1). Growing season average temperatures and precipitation sums: 1981-2003.

Table 1. Average Simulated Corn Yields (kg/ha) 1981-2003

		Applied Nitrogen in kg/ha		
		0-75	**76-150**	**151-320**
Applied irrigation water in mm	**0-70**	6955	8872	8521
	71-140	7293	9717	9100
	141-340	7275	8814	9158

Source: CropSyst simulations.

Due to settings in the crop yield simulation, the dataset contains quasi-continuous input-output combinations. In contrast to discrete application of inputs, the use of quasi-continuous input levels enables a regression approach rather than an analysis of variance. Moreover, the random application of inputs allows for unbiased estimation of the production function coefficients since input levels are uncorrelated with other variables, such as environmental factors, that also influence corn yields but are not considered in the production function estimations.

Outliers and Estimation Methodology

Exceptional climatic years are supposed to cause exceptional crop yield levels and to have an extraordinary influence on plant response to irrigation and fertilization. For instance, heat waves, droughts or waterlogged soils can indirectly restrict yield levels. Furthermore, the plants are expected to respond specifically to input management under extreme climatic conditions. As a consequence, they may involve outliers that deviate from the relationship described by the majority of the data and thus lead to a misspecification of the estimated production function.

The least squares estimator can not cope with a single outlier because one outlier can be sufficient to move the coefficient estimates arbitrarily far away from the actual underlying values. As a consequence, outliers cause unreliable coefficient estimates if LS is applied [21-24].

Two standard examples for outliers in a linear simple regression model are presented in Fig. (**2**). Point A clearly deviates from the typical linear relationship between the dependent (y) and the independent (x) variable. Such 'vertical' outlier is characterized by an unusual observation in the dependent variable. The impact of vertical outliers on the LS estimation of regression coefficients is usually small and mainly affects the regression intercept [25]. If unusual observations occur in the set of independent variables, these outliers are called leverage points. If such leverage point deviates from the linear relationship described by the majority of observations it is called 'bad leverage point' such as Point B in Fig. (**2**). Due to the exposed position of the outlier it has a leverage effect on the LS coefficient estimation. In contrast, a leverage point is called 'good leverage point' if it does not deviate from the typical relationship. Good leverage points are no outliers and even improve the regression inference as these points reduce standard errors of coefficient estimates.

Reliable regression results are provided if and only if outliers are removed or appropriately down-weighted. But, various classical methods for outlier detection, suffer from a

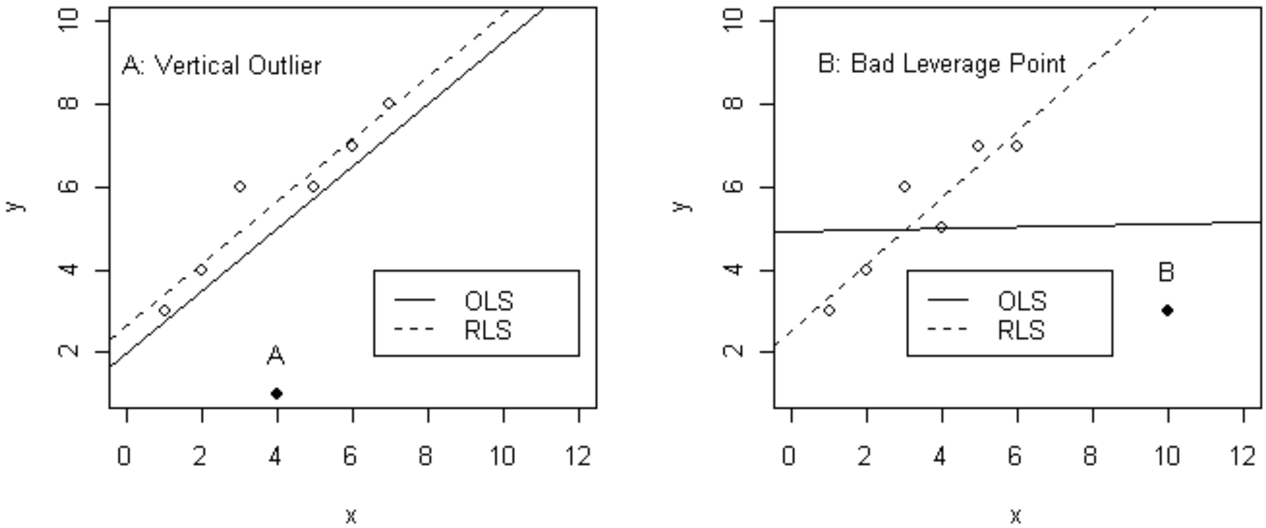

Fig. (2). Examples for outlying observations. Note: Regression lines are fitted using ordinary least squares (OLS) and reweighted least squares (RLS). Source: According to Sturm and de Haan [25].

lack of robustness [23, 24]. For instance, outliers can tilt the (original) regression line and have small regression residuals. As a consequence, outliers might not be discovered in residual plots [25]. Furthermore, studentized and jackknifed residuals, Cooks distances and other diagnostics based on Hat matrix elements, for instance, are susceptible to the so called masking effect [23]. If more than one outlier occurs, these outlier diagnostics might not be able to detect a single outlier because one outlier can be masked by the presence of others. Moreover, high dimensionality of the estimation problem and a large number of observations as it is the case for our analysis can make graphical outlier identification procedures infeasible.

In contrast, robust regression enables reliable coefficient estimation also in presence of outliers, and is therefore applied in this study. In particular, reweighted least squares (RLS) regression is used for the estimation of the quadratic and the square root production functions (equations 1 and 2). It is favored here over other robust regression methods (e.g. the MM-estimator) due to its good robustness and efficiency properties as well as because of the better interpretability of indicated outliers [23]. RLS is a weighted LS regression, which is based on an analysis of least trimmed squares (LTS) regression residuals. The LTS-estimator is a high-breakdown estimator that can cope with outlier contamination of up to 50%. Based on the idea of trimming the largest residuals the LTS fitting criterion is defined as follows:

$$\underset{\hat{\beta}}{Min} \sum_{i=1}^{h} (r^2)_{i:n} \tag{4}$$

$(r^2)_{(i)}$ are the ascending ordered squared (robust) residuals and h is the so-called trimming constant. In our analysis, $h = \left[(3n + p + 1)/4 \right]$ is employed [26], with p denoting the number of coefficients that are estimated.

The computation of LTS coefficients follows an algorithm described in Rousseeuw and Leroy [23]. Due to the low efficiency of LTS estimation, it is only used for outlier identification. An observation is identified as an outlier if the absolute standardized robust residual ($\left| r_i / \hat{\sigma} \right|$) exceeds the cutoff value of 2.5, with r_i and $\hat{\sigma}$ denoting the (robust) LTS residuals and scale estimates, respectively. This cutoff-value choice is motivated by a (roughly) 99% tolerance interval for Gaussian distributed standardized residuals [25]. With X representing the matrix of independent variables and Y the vector of the dependent variable, coefficient estimates of RLS regression are defined as follows:

$$\hat{\beta}_{RLS} = \left(X'WX \right)^{-1} X'WY \tag{5}$$

The diagonal elements of the weighting matrix ($W = diag\left\{ w_1, \ldots, w_n \right\}$) are generated by the indicator function, $I_{Outlier}$, that generates weights of zero for observations that are identified as outliers and weights of one otherwise:

$$w_i = I_{Outlier} \left[\left| \frac{r_i}{\hat{\sigma}} \right| \leq 2.5 \right] \tag{6}$$

RLS regression is applied for coefficient estimation of quasi linear functional forms, using the ROBUSTREG procedure in the SAS statistical package [27]. An example for the better robustness properties of RLS compared to LS is indicated in Fig. (**2**). LS coefficient estimates change in the presence of outliers, in particular for bad leverage points. In contrast, RLS coefficient estimates are not affected by outliers in this example.

Because LTS regression is not suitable for nonlinear problems such as the Mitscherlich-Baule function (equation 3), non-linear regression approaches are required. Robust regression is implemented in this case by using iteratively reweighted least squares (IRLS). In order to reduce the influence of outliers on estimation results, weights are generated with M-estimation using Tukey's biweight [21] such as shown in equation (7). These weights are re-estimated at each step of iteration until convergence.

$$w_i = \begin{cases} (1-(r_i/\hat{\sigma}\cdot c)^2)^2, & |r_i/\hat{\sigma}| \le c \\ 0, & |r_i/\hat{\sigma}| > c \end{cases} \tag{7}$$

r_i is the (robust) IRLS residual, $\hat{\sigma}$ the (robust) scale estimate and c a tuning constant. Following Hogg [28], we employ the median of absolute deviations from the median for robust scale estimation and set the tuning constant to 5.0. In contrast to LTS, IRLS is no high breakdown estimation technique. In order to validate results, we conduct sensitivity analysis of crucial factors such as starting values and tuning constant. We use the Levenberg-Marquardt algorithm that ensures stable estimation for highly correlated coefficient estimates that occur in our analysis [29]. In this study the nonlinear Mitscherlich-Baule function is estimated with IRLS using the NLIN procedure in the SAS software package.

Besides the most important property of giving trustworthy coefficient estimates, robust regression provides detailed insight in the structure of the data. If LS and robust regression results are considerably different and many outliers are identified, the related observations have to be examined. Above all, the interpretation of outliers is indispensable. Ruling out that outliers are caused by typing, copying or measuring errors, this interpretation should take not only statistical but mainly reasons from the agronomic point of view into account. Thus, in the following, all estimations are conducted with both least squares and robust regression and outlier interpretation is provided.

RESULTS

Estimation Results

Within our dataset, the year 2003 involves the largest number of observations identified as outliers. About 25% of the observations that are identified by the RLS method as outlier or are given very small weights in the IRLS method, can be attributed to this particular year[1]. It is characterized by high temperatures and low precipitation in the relevant seeding-to-harvest period that caused particularly low corn yields in all Europe [30]. Other years with exceptionally low levels of precipitation and high temperatures during the corn growing season (e.g. 1983, 1991) also frequently occur in the lists of outlying observations.

The reason for the existence of outliers in these years is twofold. First, the yield levels are lower than usually. Second, the relationship between independent and dependent variables is affected by different reactions to input levels in situations where one of the inputs is a limiting factor. The yield response to irrigation water is higher than usual if - unlike in normal years - water constitutes a limiting factor for the plants in the Swiss Plateau. Furthermore, the interaction between fertilizer and irrigation water is higher because the plants' response to nitrogen also highly depends on water availability as nitrogen is taken up by the roots in a water solution.

[1] In total RLS identifies 43 outliers for the quadratic production function and 37 for the square root function. Moreover, 36 observations have weights smaller than 0.25 in the IRLS estimation of the Mitscherlich-Baule function.

Table **2** presents the estimation results for the quadratic and the square root production functions, respectively. The coefficient α_5 (Applied Nitrogen * Irrigation Water) is not significantly different from zero in the four estimated polynomial functions. This indicates that rainfall is sufficient to ensure efficient nitrogen uptake under normal climatic conditions in Switzerland.

Table 2. Coefficient Estimates for the Quadratic and the Square Root Production Functions

Variable	Least Squares	Reweighted Least Squares
	Quadratic production function (equation 1)	
Intercept	6638.27 (165.05)**	6661.42 (179.24)**
N	25.64 (17.62)**	27.55 (22.71)**
W	6.05 (5.62)**	5.58 (5.75)**
N^2	-0.071 (12.22)**	-0.0724 (14.94)**
W^2	-0.018 (3.87)**	-0.0162 (3.88)**
NW	0.0078 (1.51)	0.0037 (0.89)
adj. R^2	0.57	0.71
	Square root production function (equation 2)	
Intercept	6589.99 (155.02)**	6601.92 (162.13)**
$N^{1/2}$	297.18 (12.42)**	313.09 (16.34)**
$W^{1/2}$	75.09 (4.26)**	67.14 (4.17)**
N	-11.22 (6.88)**	-10.54 (8.15)**
W	-3.03 (2.40)*	-2.5 (2.17)*
$(NW)^{1/2}$	1.46 (1.43)	0.36 (0.45)
adj. R^2	0.58	0.73

Note: Statistics in parentheses are t statistics. (**) and (*) indicates significance at the 1% and 5% level, respectively.

The Mitscherlich-Baule production function estimates are presented in Table **3**. It shows that the coefficient estimates for irrigation water and water endowment (α_4 and α_5) are not significantly different from zero at the level of five percent in the LS estimation. In contrast, the coefficients α_4 and α_5 are significant at the one percent level if robust regression (IRLS) is used. Moreover, the coefficient estimate for α_5 increases remarkably if IRLS regression is applied. This is explained by the fact that mainly dry years are excluded or down-weighted in the robust regression, such that the estimated soil water endowment is higher for the remaining observations.

Yet, the decision on the most appropriate estimation technique cannot exclusively be based on statistical measures. For instance, the goodness of fit cannot be compared between different estimations because the deletion of outliers, by definition, increases the goodness of fit for the regression on the remaining observations. Hence, conclusions on the appropriateness of functional forms and estimation techniques can be drawn if and only if the misspecification costs are calculated and interpreted, as shown in the subsequent section.

Table 3. Coefficient Estimates for the Mitscherlich-Baule Production Function

Variable	Least Squares	Iteratively Reweighted Least Squares
α_1	9180.6 (95.14)**	9410.3 (87.7)**
α_2	0.0288 (5.72)**	0.0266 (7.38)**
α_3	50.6952 (5.96)**	48.3036 (7.75)**
α_4	0.0598 (1.22)	0.0304 (2.95)**
α_5	45.14 (1.24)	71.22 (3.10)**
adj. R^2	0.74	0.81

Note: Statistics in parentheses are t statistics. (**) indicates significance at the 1% and 5% level, respectively.

Optimal Input Levels and Costs of Misspecification

The analysis of production functions usually involves an assessment of optimal input and output levels, which is generally determined through maximization of a suitably defined objective function. For the purpose of our analysis, this is given by the subsequent profit function

$$\pi = P_{Corn} \cdot f(W,N) - P_{Nitrogen} \cdot N - P_{Irrigation} \cdot W \tag{8}$$

where the net return (or quasi-rent) per hectare π is equal to the gross return (crop price P_{Corn} times corn yield $f(W,N)$), minus total nitrogen costs (nitrogen price $P_{Nitrogen}$ times amount of nitrogen applied N) and total irrigation costs (irrigation price $P_{Irrigation}$ times amount of irrigation water W) per hectare. For simplicity, other costs are assumed to be constant and therefore irrelevant for calculating the profit maximizing input combination. By maximizing the above profit function (equation 8), the optimal input levels are determined through the following first-order conditions, where N^* and W^* are the profit maximizing input levels of nitrogen fertilizer and irrigation water, respectively:

$$\frac{\partial f(W,N^*)}{\partial N} = \frac{P_{Nitrogen}}{P_{corn}} \text{ and } \frac{\partial f(W^*,N)}{\partial W} = \frac{P_{Irrigation}}{P_{corn}} \tag{9}$$

These conditions are satisfied if the input price equals the value marginal product of each production factor; i.e., the crop price multiplied by the factor's marginal productivity for each input factor.

In the further analysis, we set the corn price equal to CHF 0.642 kg^{-1}, the average annual value for the period 1981-2003 in Switzerland [31]. We assume a nitrogen price of CHF 1.6 kg^{-1} [32], and a price for irrigation water of CHF 0.06 m^{-3} [20]. Using these data, the optimal input levels are calculated according to equation (9) and represented in Table **4**.

It shows that all optimal input levels are within the range of the data. With 61.3 mm of irrigation water and 111.2 kg/ha of nitrogen, the lowest input use is recommended by the Mitscherlich-Baule function estimated with LS. This goes along with the lowest yield (9078 kg/ha) and an estimated net revenue of 5613.55 CHF/ha. In contrast, the robust estimated quadratic function shows the highest yield (9859 kg/ha) and nitrogen use (177.4 kg/ha) and the highest profit (5947.68 CHF/ha), while the quadratic LS function implies the highest optimal amount of irrigation water with 179.6 mm. Thus, the quadratic form implies a higher optimal use of nitrogen and irrigation water than all other functions. This confirms with the evidence given by Anderson and Nelson [6] regarding the overestimation of optimal input amounts by the quadratic form.

Furthermore, the results in Table **4** show that the robust versions of production function estimates systematically lead to higher profit maximizing yields and higher profits than their non-robust counterparts. Moreover, for each functional form, the optimal amount of nitrogen fertilizer application increases if robust regression results are taken instead of LS results. And, except for the case of the Mitscherlich-Baule function, robust regression leads to the expected adjustment towards lower use of irrigation water in the profit maximizing situation.

It shows that the range of optimal input levels is much wider for LS than for robust regression. This indicates that differences in optimal input recommendation are not only caused by differences in the analyzed functional forms but also caused by the effect of outliers on LS estimation. All in all, the use of robust estimation narrows the range of optimal input levels across the different functional forms. Thus, the application of robust regression to production function estimation reduces the differences between different functional forms.

Table 4. Optimal Input Levels, Yield, and Maximum Net Return

Functional Form-Estimation Method	Optimal Amount of Nitrogen Applied (kg/ha)	Optimal Amount of Irrigation Water Applied (mm)	Optimal Yield (kg/ha)	Maximum Net Return (CHF/ha)
Quadratic-LS	172.8	179.6	9695	5840.32
Square Root-LS	131.3	133.9	9180	5602.82
Mitscherlich-Baule-LS	111.2	61.3	9078	5613.55
Quadratic-RLS	177.4	163.8	9859	5947.68
Square Root-RLS	147.7	108.6	9324	5684.56
Mitscherlich-Baule-IRLS	124.9	116.7	9286	5691.51

Note: LS indicates least squares, RLS reweighted least squares, and IRLS iteratively reweighted least squares estimation.

Table **4** shows furthermore that the selection of the functional form and the selection of the estimation method both affect the result of the economic optimization and allocation problem. This relates to the concept of misspecification costs, which we employ for the final evaluation of production functions and estimation methods. The relative costs of misspecification are defined as the decrease in net return if optimal input levels of an incorrect function are used instead of those of the real underlying production function. With this concept, the potential loss of a misspecification of the production function is minimized. Usually, the focus is on the potential loss due to the wrong functional form. In the following, we also consider the costs of using the improper estimation technique.

Table **5** gives the relative costs of misspecification. The nine cells in the upper left-hand corner correspond to the traditional approach where only functional forms estimated with LS are compared. If for instance the quadratic function would be the true underlying form, the use of the square root function induces a cost of misspecification of CHF 93.01. For the Mitscherlich-Baule function, this increases to CHF 297.88. The latter exhibits the highest costs of misspecification, while the square root function is the most appropriate if the misspecification-cost criterion is employed.

The square root function is similar to the quadratic form, but flatter in its surface and comes therefore closer to the plateau approach of the Mitscherlich-Baule specification [1]. Optimal input recommendations based on the square root function are correspondingly situated between those of the other two approaches we consider here.

Table **5** further reveals that, in most cases, the use of robust estimation methods results in lower costs of misspecification than the standard LS approach, and that the square root specification performs better under this criterion than the other functional forms. This can be seen when comparing the top left-hand cells with the bottom right-hand ones, as well as from the comparison of the misspecification costs in the different lines of Table **5**. Only in the cases where the square root specifications are assumed to be the true underlying functions does the quadratic LS estimation show slightly lower costs of misspecification than its RLS counterpart. Furthermore, square root function estimation with LS leads to a marginally lower decrease of the net profit than its robust counterpart if the Mitscherlich-Baule-LS is assumed to be the underlying function. Altogether, this supports the suggestion that the RLS estimation of the square root function is the best approximation of the here analyzed crop response relationship with regard to nitrogen fertilization and irrigation.

SUMMARY AND CONCLUSIONS

The improved estimation of production functions might be valuable in practice because crop production functions are widely applied, for instance, to assess agro-environmental policy measures [13] to compare cropping systems [12] or to project future agricultural water demand [16].

In our study, simulated corn yield data for the Swiss Plateau are used for the estimation of crop production functions, with particular consideration of yield response to nitrogen fertilizer and irrigation water application. Three functional forms are considered: the quadratic, the square root, and the Mitscherlich-Baule function. In addition, robust and standard regression methods are used for the estimation.

We found the square root function to be the most appropriate form to represent the data generated with corn yield simulations for Switzerland. Furthermore, exceptional climatic events, such as the summer drought in 2003, are proved to be the major source of misleading results if the least squares criterion is used to estimate production function coefficients. Robust regression methods are recommended instead. The use of robust estimation narrows the range of optimal input levels across the different functional forms. Thus, differences between functional forms are reduced by applying robust regression. This conclusion is further supported by a comparison of the relative costs of misspecification. Using robust instead of least squares regression generally results in lower costs of misspecification. Irrespective of the true underlying functional form, optimal input levels based on robust estimated functions reduce the maximum costs of misspecification compared to the counterparts estimated with least squares regression. Thus, our investigation shows that, besides the functional form, the estimation method is decisive for production function comparisons.

This is even more important for climate change related questions. Climate - and thus crop yield - extreme events are expected to occur more often in the future due to climatic change [e.g. 33]. The properties of robust regression to ensure efficient and reliable coefficient estimation in presence

Table 5. Relative Costs of Misspecification

	Cost (in CHF/ha) of using Optimal Input Levels Based on:					
When the True Function is:	**Quadratic-LS**	**Square Root-LS**	**Mitscherlich-Baule-LS**	**Quadratic-RLS**	**Square Root-RLS**	**Mitscherlich-Baule-IRLS**
Quadratic-LS	0	93.01	297.88	4.23	77.85	135.18
Square Root-LS	30.61	0	39.83	32.13	8.41	2.01
Mitscherlich-Baule-LS	113.22	41.38	0	109.97	41.86	27.34
Quadratic-RLS	3.77	104.65	296.39	0	68.59	145.23
Square Root-RLS	7.18	27.08	35.49	8.45	0	23.14
Mitscherlich - Baule-IRLS	57.52	54.08	3.11	51.85	9.86	0

Note: LS indicates least squares, RLS reweighted least squares and IRLS iteratively reweighted least squares estimation.

of outliers might thus be particularly valuable for applications and economic assessments related to climate change issues [e.g. 14].

Altogether, robust regression is a valuable tool for a wide range of modeling problems that require a proper representation of crop response functions to variable inputs, such as nitrogen fertilizer and irrigation water. Further research should apply other data sets and other robust regression methods, such as MM-estimation, to validate the here presented results. Moreover, in a further step of economic analysis, the observations that are identified as outliers should be re-incorporated in the optimization model. Regression residuals from production function estimation can be used to estimate yield variation with respect to input use. Production and yield variation functions can then be integrated into a utility maximization model that augments the here presented profit maximization approach [e.g. 34]. Thus, the application of robust regression can improve the estimation of both production and yield variation functions.

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ABBREVIATIONS

CHF	=	Swiss Francs
IRLS	=	Iteratively reweighted least squares
LS	=	Least squares
LTS	=	Least trimmed squares
RLS	=	Reweighted least squares

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